

15)  $\lim_{x \rightarrow 3^+} \frac{x}{x-3}$

(2) sign?

1) substitution?

$\Rightarrow \frac{3_{ish}}{0_{ish}}$

$+\infty$

$-\infty$

DNE

5, 4, 3.1, 3.00001, ...

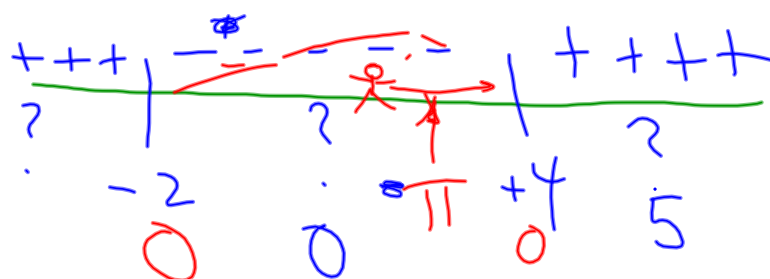
(+)

2.2)  
25

$$\lim_{x \rightarrow 4^-} \frac{3-x}{x^2-2x-8} = \frac{(3-x)}{(x-4)(x+2)} \quad \text{as } x \rightarrow 4^- \rightarrow +\infty$$

zeros at  $x=+4, -2$

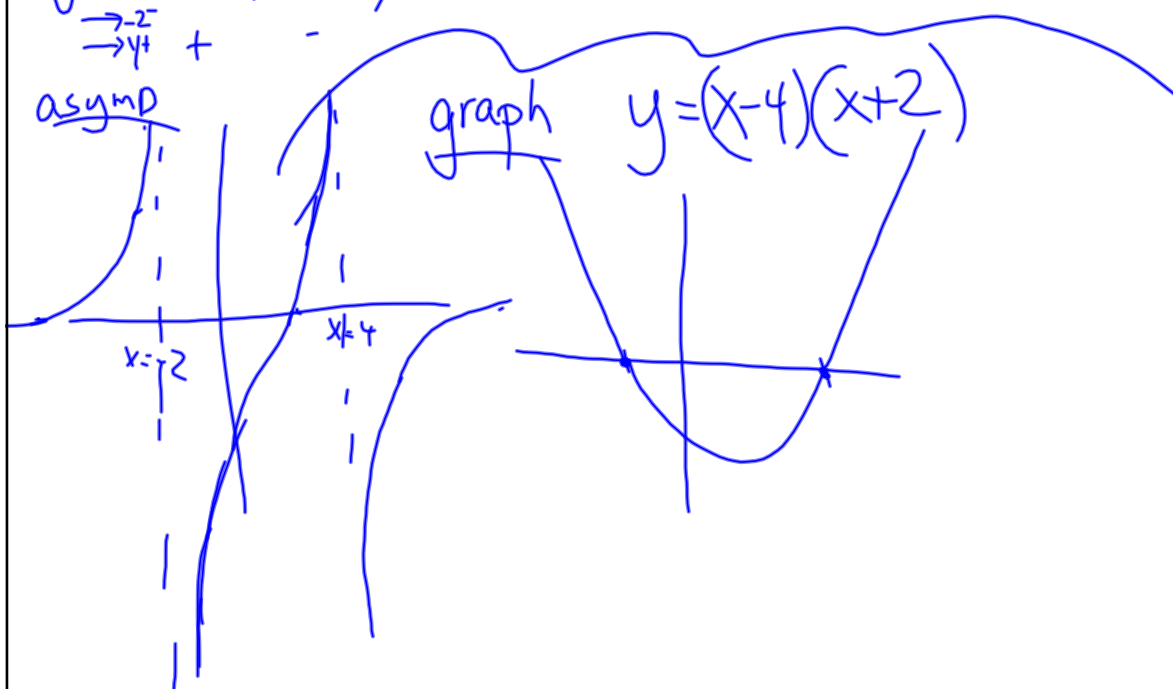
(2) sign



$$y = (x-4)(x+2)$$

 $\begin{matrix} \rightarrow -2 \\ \rightarrow 4 \end{matrix}$ 

asympt

graph  $y = (x-4)(x+2)$ 

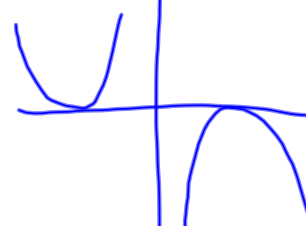
Multiplicities → the exponent of the factor of the polynomial

A root/factor

w/ multiplicity = 1  
⇒ cut x-axis



w/ even multiplicity  
exp = 2 or 4 or ...  
⇒ touch x-axis then chg direct



w/ odd multiplicity  $\geq 3$   
exp = 3, 5, 7  
level out @ x-axis then cross over



2.11  
27)

$$\lim_{x \rightarrow 2^+}$$

$$\frac{1}{|2-x|} = +\infty$$

Always  
treat abs.  
value  
thusly:

$$|2-x| = \begin{cases} 2-x & \text{when } 2-x \geq 0 \\ & 2 \geq x \\ -(2-x) & \text{when } 2-x < 0 \\ & 2 < x \end{cases}$$

$$|x| = \begin{cases} x & (\text{when } x \geq 0) \\ -x & (\text{when } x < 0) \end{cases}$$

$$|2-x| = \begin{cases} 2-x & \text{when } x \leq 2 \\ x-2 & \text{when } x > 2 \end{cases}$$

$$\lim_{x \rightarrow 2^+} \frac{1}{x-2} = +\infty$$

$$\frac{1}{0} \begin{cases} +\infty \\ -\infty \\ \text{DNE} \end{cases}$$

$\lim_{x \rightarrow 2^+} \frac{-1}{(x-2)(x+1)}$   $\rightarrow -\infty$   
 sign of  $\frac{-1}{(x-2)(x+1)}$   $\begin{array}{c|ccc} & - & - & + \\ \hline x & -2 & -1 & 0 \\ \hline & + & + & - \end{array}$

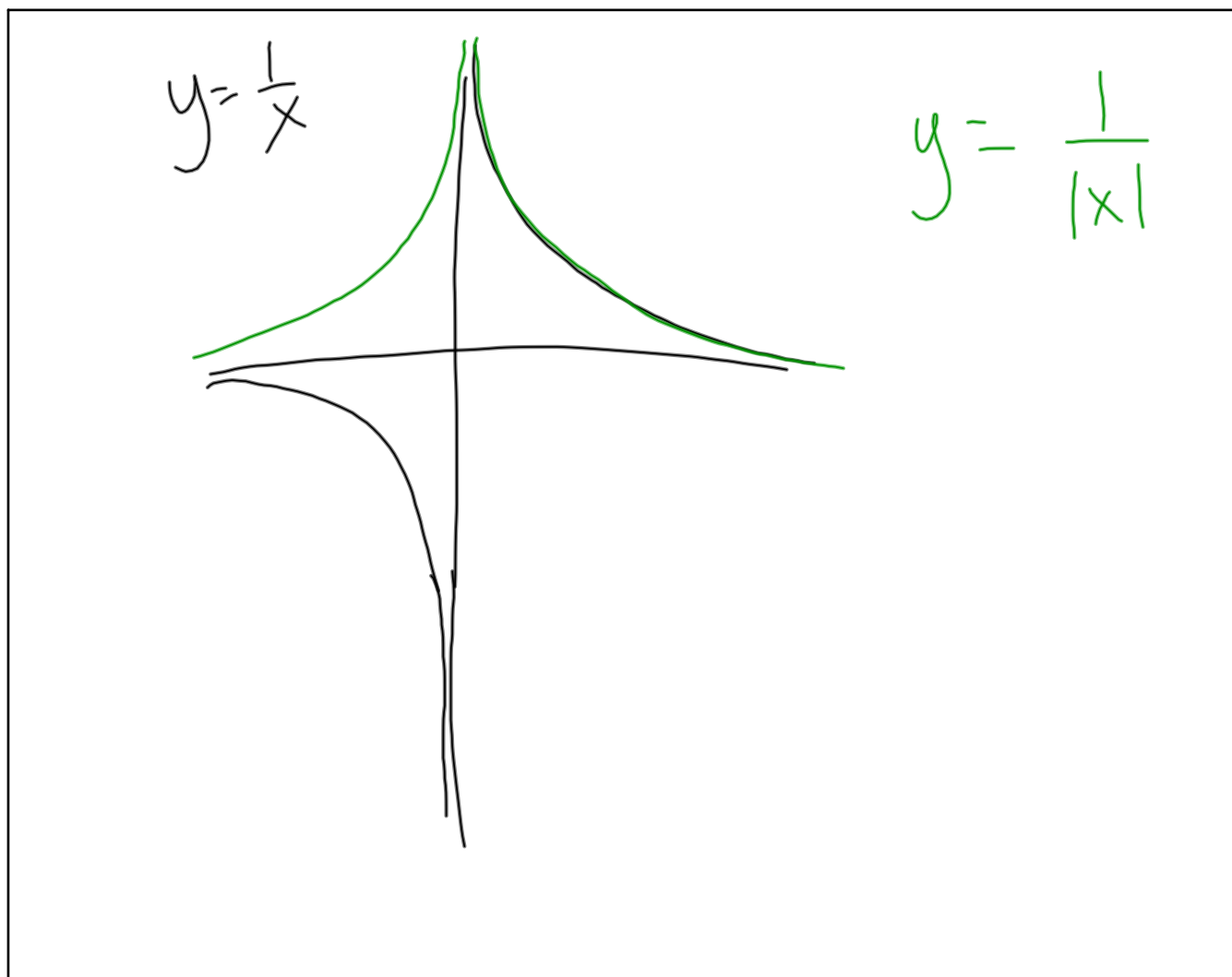
$\lim_{x \rightarrow 2^+} \frac{-1}{(x-2)^2(x+1)}$   $\rightarrow -\infty$   
 sign of  $\frac{-1}{(x-2)^2(x+1)}$   $\begin{array}{c|ccc} & - & - & + \\ \hline x & -2 & -1 & 0 \\ \hline & + & + & - \end{array}$

$\lim_{x \rightarrow 2^-} \frac{-1}{(x-2)(1-x)}$   $\rightarrow +\infty$   
 sign of  $\frac{-1}{(x-2)(1-x)}$   $\begin{array}{c|ccc} & - & - & + \\ \hline x & -2 & -1 & 0 \\ \hline & + & + & - \end{array}$

$$\lim_{x \rightarrow 2} \frac{-1}{(x-2)^2(x+1)} = -\infty \text{ because}$$



$$\lim_{x \rightarrow 2^+} \frac{-1}{(x-2)^2(x+1)} = \lim_{x \rightarrow 2^-} \frac{-1}{(x-2)^2(x+1)} = -\infty$$



2.1/18

i  $f(x)=1$  if  $x$  is an integer  $>0$

$f(x) \neq 1$  if  $x$  is not an int.  $>0$

ii  $f(x)=-1$  if  $x$  is neg integer

not  $= -1$  if  $x$  is not an int  $<0$

iii  $\lim_{x \rightarrow +\infty} f(x) = 1$

$\lim_{x \rightarrow -\infty} f(x) = -1$

horizontal asymptotes

